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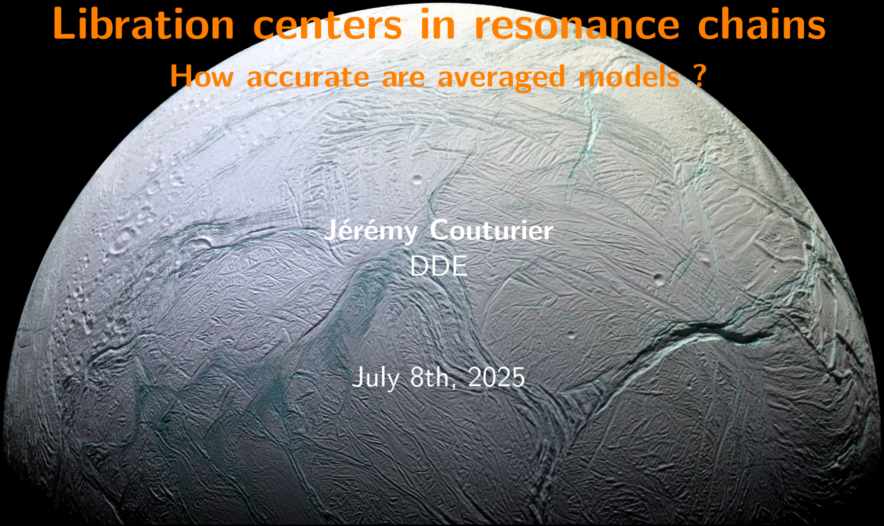

Libration centers in resonance chains

How accurate are averaged models ?

Jérémy Couturier
DDE

July 8th, 2025

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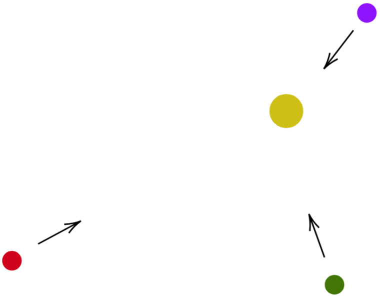
Planetary systems do not have equilibria



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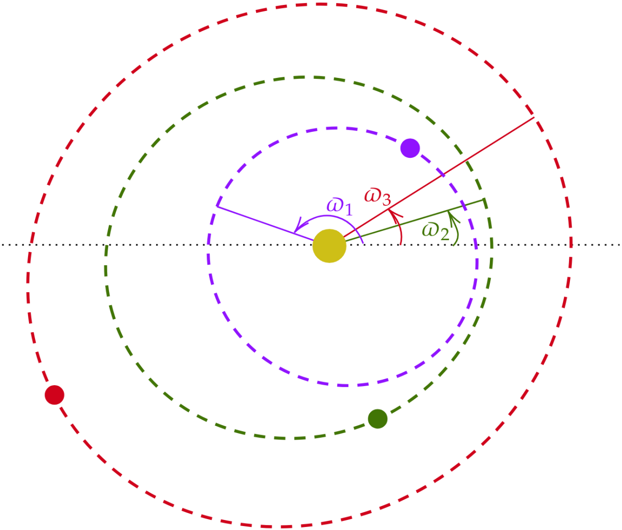
Equilibria appear when considering the orbits only



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At an equilibrium:

- λ_j are not constant $\rightarrow$ planets keep orbiting
- a_j, e_j, i_j are constant (are variables of the Hamiltonian)
- ϖ_j, Ω_j are not constant (are not variables of the Hamiltonian)

Only angles invariants by rotation are explicit variables of the Hamiltonian:

- $\varpi_1 - \varpi_2$ and $2\varpi_1 - \varpi_2 - \varpi_3$ are constant angles
- $p_1\varpi_1 + p_2\varpi_2 + p_3\varpi_3$ is constant if $p_1 + p_2 + p_3 = 0$

The ϖ_j must all precess at the same frequency

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What about a real system?

Averaged models have many approximations:

- Model limited to low-order in eccentricity
- Perturbative part evaluated at nominal semi-major axes
- Averaging over the fast angles of the Hamiltonian
- Contributions in $\mathcal{O}(m_j/m_0)^2$ disregarded

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Fixed points vs libration center ? Chain $3:4:6:8$

Fixed points

- $\phi_1 = \lambda_1 - 2\lambda_2 + \lambda_3 =$ Cte
- $\phi_2 = \lambda_2 - 3\lambda_3 + 2\lambda_4 =$ Cte
- $\phi_3 = \lambda_3 - \lambda_4 \to$ fast circulating at frequency ν_3
- $\phi_4 = -3\lambda_3 + 4\lambda_4 \to$ precessing with the ϖ_j at frequency ν_4
- $\sigma_j = -3\lambda_3 + 4\lambda_4 - \varpi_j =$ Cte

Libration center

- $\phi_1 = \lambda_1 - 2\lambda_2 + \lambda_3 =$ is periodic of period ν_3
- $\phi_2 = \lambda_2 - 3\lambda_3 + 2\lambda_4 =$ is periodic of period ν_3
- $\phi_3 = \lambda_3 - \lambda_4 = \nu_3 t +$ periodic of period ν_3
- $\phi_4 = -3\lambda_3 + 4\lambda_4 = \nu_4 t +$ periodic of period ν_3
- $\sigma_j = -3\lambda_3 + 4\lambda_4 - \varpi_j =$ is periodic of period ν_3

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I am creating a software that

- Automatically studies a given chain $p_1 : p_2 : \cdots : p_n$
- Can manipulate both the averaged and complete Hamiltonian
- Can find fixed points
- Can find *libration centers*

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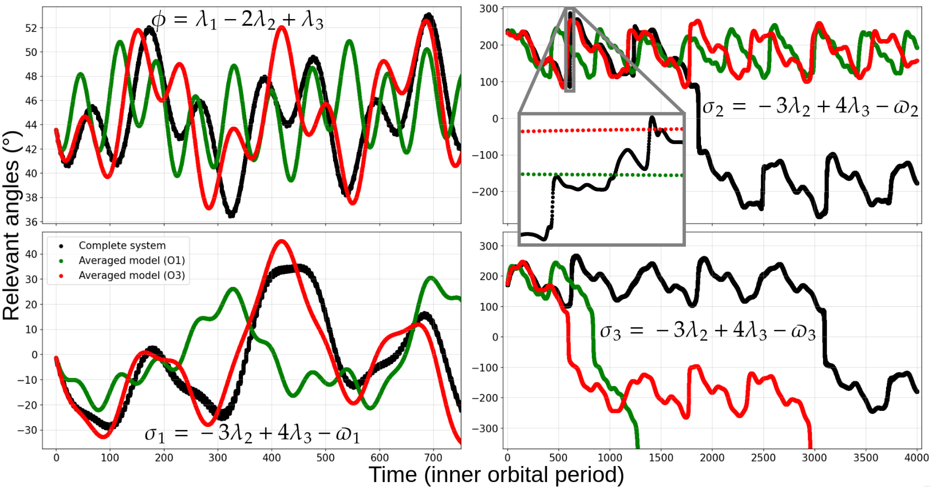
Comparison averaged model and complete system for Kepler-60



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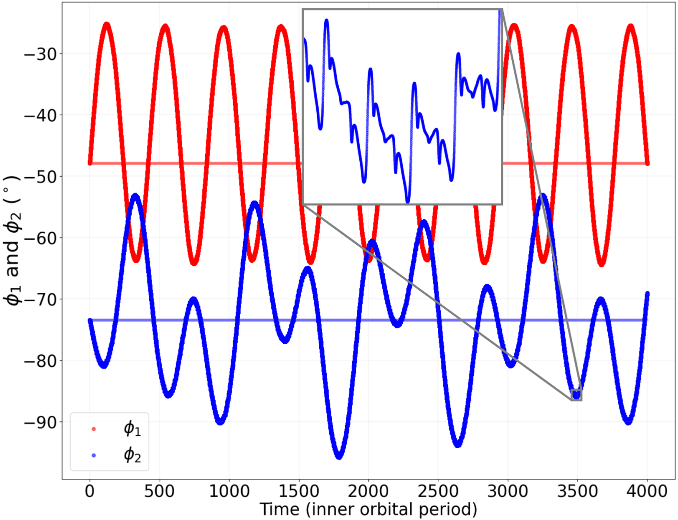
Integrating at a fixed point of the model (Chain $3 : 4 : 6 : 8$)



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Aptidal can find libration centers

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Starting the search for a libration center.

Iteration n° 0 : (lbd_1, lbd_2, lbd_3) = (1.25334744208060, -1.45983490094329, 2.89556622661035)
                 (vrp_1, vrp_2, vrp_3) = (0.25380634132068, 3.39539899491113, 0.25380634132261)
                 (a_1, a_2, a_3)       = (0.99998852270189, 1.16286469253592, 1.41375862174436)
                 (e_1, e_2, e_3)       = (0.002454540981114477, 0.005159761514032148, 0.001524850709017595)

Iteration n° 1 : (lbd_1, lbd_2, lbd_3) = (3.33330798274093, 5.26002118304277, 1.65227298241955)
                 (vrp_1, vrp_2, vrp_3) = (0.47259744136410, 3.55948908857327, 0.32884036430858)
                 (a_1, a_2, a_3)       = (0.99995604711164, 1.16295562399184, 1.41371348167575)
                 (e_1, e_2, e_3)       = (0.002217198276845951, 0.004933795394239858, 0.001586720532824100)

nu_2 = dphi_2/dt = 1.27236541724418, nu_3 = dphi_3/dt = -0.07876080736703, nu_2/nu_3 = -16.15480414408320
n_1/n_2 = 1.25404770, n_2/n_3 = 1.34040046
Amplitude = 0.00010614976100192660, required = 0.0000001523192

Iteration n° 2 : (lbd_1, lbd_2, lbd_3) = (5.34223606186828, 5.60944294599532, 0.34354297783725)
                 (vrp_1, vrp_2, vrp_3) = (0.54368907202440, 3.63009653651983, 0.38955128015460)
                 (a_1, a_2, a_3)       = (1.00006341837660, 1.16268722337893, 1.41381753291775)
                 (e_1, e_2, e_3)       = (0.002141230582097681, 0.004841700091519936, 0.001590312941810518)

nu_2 = dphi_2/dt = 1.27236445995839, nu_3 = dphi_3/dt = -0.07875696205510, nu_2/nu_3 = -16.15558074812828
n_1/n_2 = 1.25404551, n_2/n_3 = 1.34040446
Amplitude = 0.00000142053828396651, required = 0.0000001523192

Iteration n° 3 : (lbd_1, lbd_2, lbd_3) = (1.06334414862739, 5.95784468358055, 5.31723326140722)
                 (vrp_1, vrp_2, vrp_3) = (0.31221360091268, 3.36623503005152, 0.04694359448298)
                 (a_1, a_2, a_3)       = (1.00000000000000, 1.16301343913759, 1.41385484266288)
                 (e_1, e_2, e_3)       = (0.002448203293674203, 0.004917651499622591, 0.001399807102696927)

nu_2 = dphi_2/dt = 1.27236439623561, nu_3 = dphi_3/dt = -0.07875671810627, nu_2/nu_3 = -16.15562998090792
n_1/n_2 = 1.25404549, n_2/n_3 = 1.34040445
Amplitude = 0.00000000107100540944, required = 0.0000001523192
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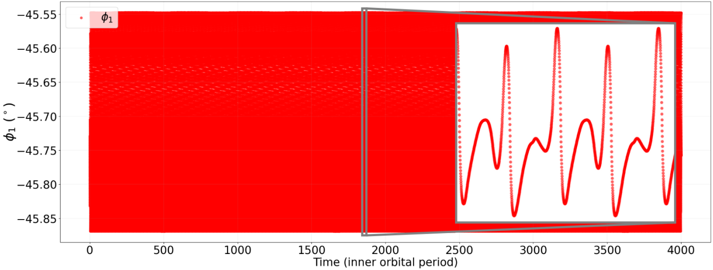
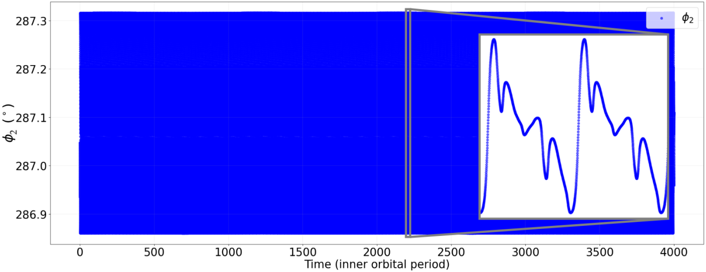
Libration center





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Case of Kepler-60

- Using the medians of Leleu's default posterior
- $(m_1, m_2, m_3) = (0.00001396343, 0.00001018846, 0.00001168183)$
- $(a_1, a_2, a_3) = (1, 1.16051594624160, 1.40732829159100)$
- $(e_1, e_2, e_3) = (0.002443205783, 0.038988792391, 0.001929629299)$
- $(\lambda_1, \lambda_2, \lambda_3) = (1.22847960195, -1.45983490094, 2.89556622661)$
- $(\varpi_1, \varpi_2, \varpi_3) = (-0.67780162957, -2.86441121138, 0.35647438423)$

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Kepler-60 has a family of libration centers parameterized by G

On this family, I looked for the libration center that:

- Has the same value of G as the median of Leleu's posterior
- Has period ratios n_1/n_2 and n_2/n_3 closest to those of Leleu's posterior

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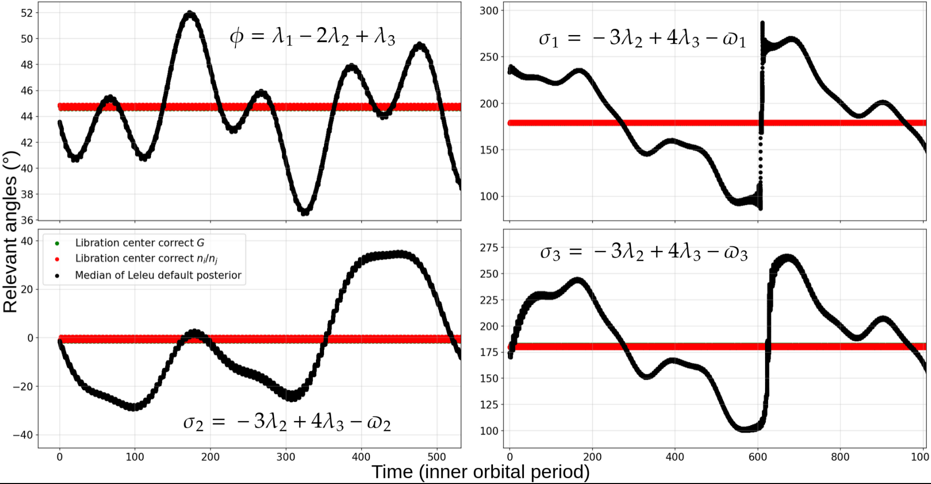
Observatoire de Genève, fondé en 1772

Kepler-60 with two possible libration centers



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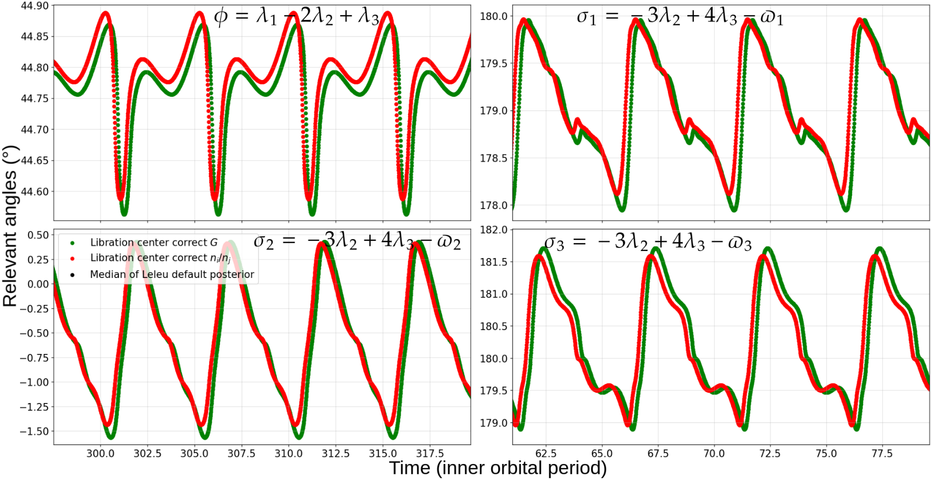
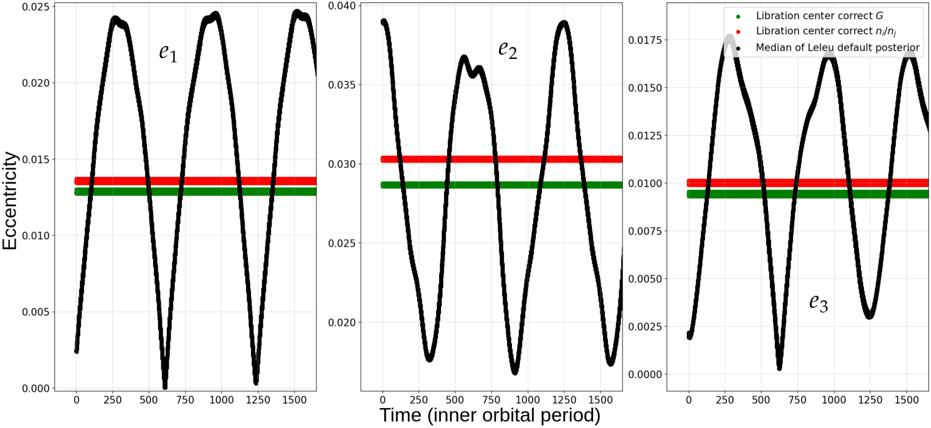
Both libration centers are close to each other



Kepler-60 with two possibles libration centers



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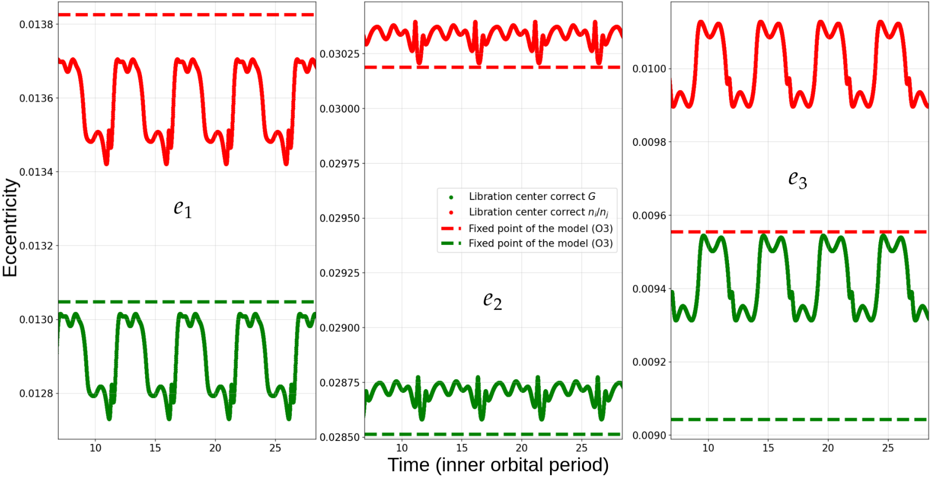
The model (at order 3) is rather accurate



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Kepler-60 could have converged towards one of them through tides

Possible state for Kepler-60 assuming convergence towards a libration center:

- $(m_1, m_2, m_3) = (0.00001396343, 0.00001018846, 0.00001168183)$
- $(a_1, a_2, a_3) = (1, 1.16084035919950, 1.40671871088901)$
- $(e_1, e_2, e_3) = (0.013499025712, 0.030339683494, 0.010088416895)$
- $(\lambda_1, \lambda_2, \lambda_3) = (5.62613039777, 0.78403931790, 3.00767556394)$
- $(\varpi_1, \varpi_2, \varpi_3) = (0.27785699708, 3.41403973628, 0.26235900508)$

Thank you for your attention