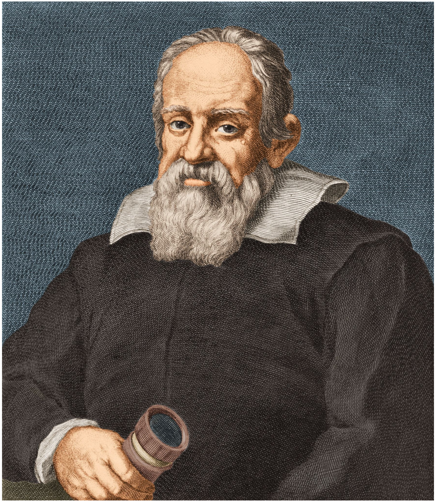
A Journey into Saturn's rotation

Why is Saturn tilted ?

Jérémy Couturier

March 3rd, 2023

First awareness of planet's rotation

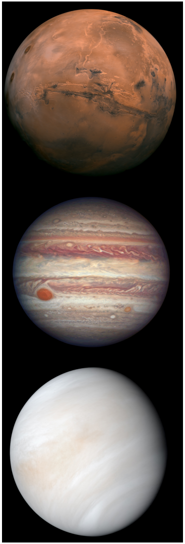


Galileo Galilei

Giovanni Domenico Cassini

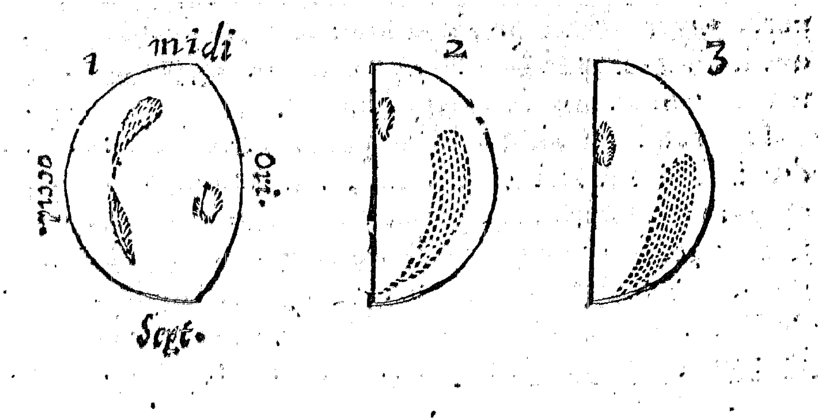
First measurements (1665-1667)

Giovanni Domenico Cassini



- Mars
  $24\,\text{h}\,40'$
- Jupiter
  $9\,\text{h}\,56'$
- Venus
  $\lesssim 24\,\text{h}$

Grasping Venus' rotation is not easy



Observation of Venus by G. D. Cassini in February and April 1667.

"Until the 28th day of April (1667), I could not notice any shiny spot similar to that which I had already seen."

Further measurements for Venus

- 1667: Cassini $\rightarrow \lesssim 24\,\mathrm{h}$
- 1726: Bianchini $\rightarrow 24$ days $8\,\mathrm{h}$
- 1740: Cassini (son) $\rightarrow 23\,\mathrm{h}\ 20'$
- 1740: Schroeter $\rightarrow 23\,\mathrm{h}\ 20'\ 19''$
- 1842: De Vico $\rightarrow 23\,\mathrm{h}\ 21'\ 21.9345''$

Cassini

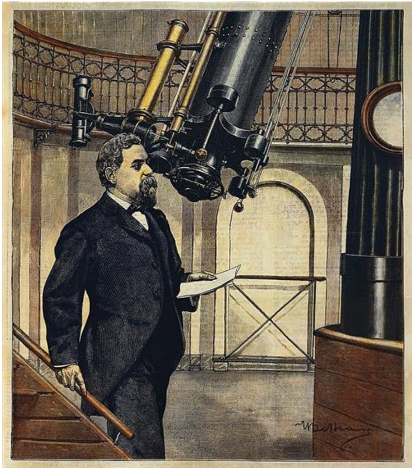
Bianchini

Cassini son

Schroeter

De Vico

Giovanni Schiaparelli (1835-1910)

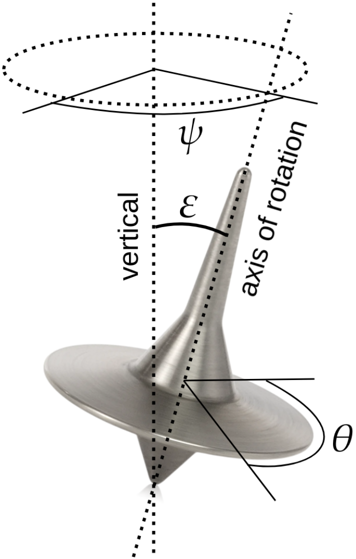


IL GRANDE ASTRONOMO SCHIAPARELLI NELL' OSSERVATORIO DI BRERA.
(Disegno di A. Beltrame, dal vero).

- 1884: Georges Darwin
  Tidal forces $\rightarrow$ Synchronous rotation of the Moon
- 1889: Schiaparelli
  Cartography of Mercury's surface $\rightarrow$ Synchronous rotation (~ 88 days)
- 1890: Schiaparelli
  Venus should be synchronous to the Sun $\rightarrow 224.7$ days
- 1965: Jet Propulsion Laboratory
  Radar measurement of Mercury
  $3:2$ resonance (~ 59 days)
- 1964: Goldstein & Carpenter
  Radar through Venus' clouds $\rightarrow$
  -243 days !!

What about a top ?

Euler angles for solid rotation description



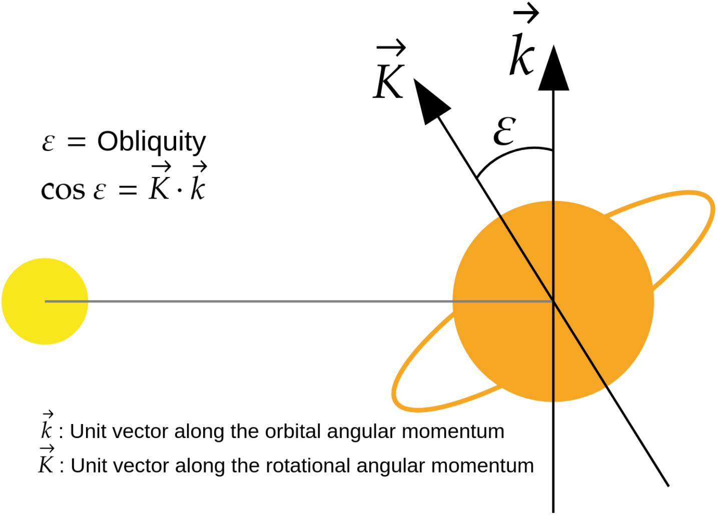
θ : Spin angle
ψ : Precession angle
ε : Obliquity or tilt

$$\boxed{\dot{\theta} \gg \dot{\psi} \gg \dot{\varepsilon}}$$

If a top does it, why not a planet ?

ε = Obliquity

$\cos\varepsilon = \vec{K} \cdot \vec{k}$



$\vec{k}$: Unit vector along the orbital angular momentum

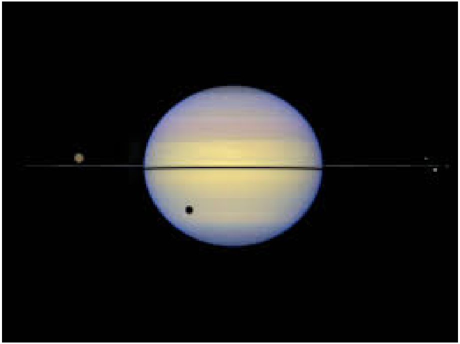
$\vec{K}$: Unit vector along the rotational angular momentum

Obliquities in the Solar system

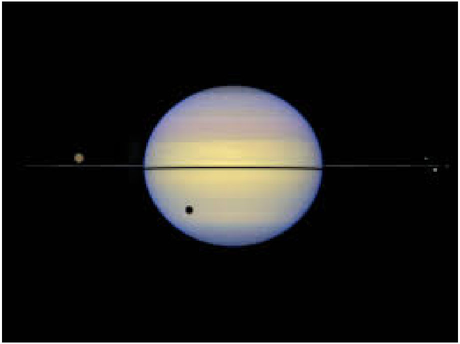
Planet	Obliquity
Mercury	$0.034°$
Venus	$177.4°$
Earth	$23.44°$
Mars	$25.19°$
Jupiter	$3.13°$
Saturn	$26.73°$
Uranus	$97.77°$
Neptune	$28.32°$

- Laskar & Robutel (1993):
  The inner planets' obliquities were chaotic at some point $\rightarrow$ Not primordial.
- Planets are expected to form with near-zero obliquity.
- The outer planets' obliquities should be primordial but only Jupiter's is
- Why is Saturn so tilted ?

What do a top and Saturn have in common ?

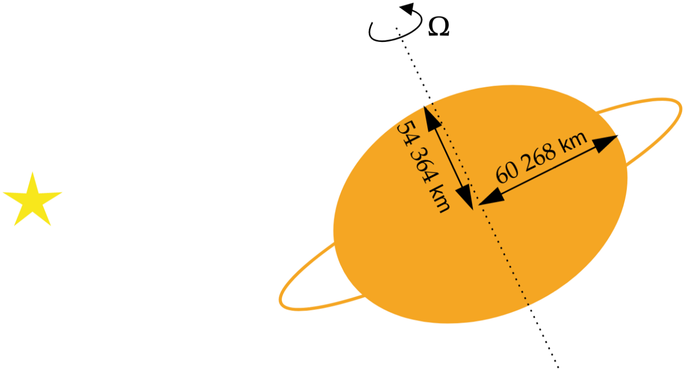


What do a top and Saturn have in common ?



An equatorial bulge !

Equatorial bulge



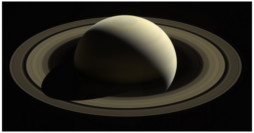
- Saturn flattens under its quick rotation → equatorial bulge
- The top was built with an equatorial bulge
- Saturn precesses because the Sun pulls on its equatorial bulge
- The top precesses because the Earth pulls on its equatorial bulge

Precession frequency of the top

$$\dot{\psi} = \frac{2Mgl}{C\dot{\theta}} \cos\varepsilon$$

Precession frequency of Saturn

$$\dot{\psi} = -\frac{3n^2 M R^2}{2C\dot{\theta}} J_2 \cos\varepsilon$$



- The fastest the spin, the slowest the precession
- The largest the tilt, the slowest the precession

Precession of Saturn

The corresponding period of the precession of the equinox is

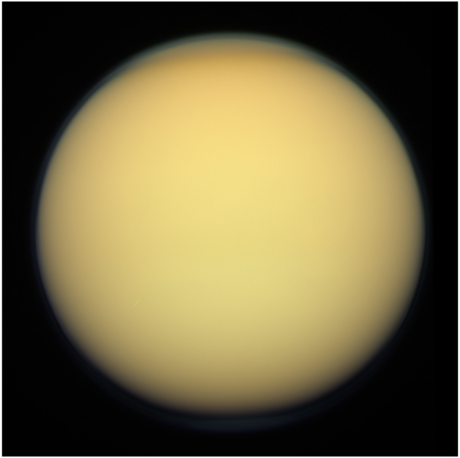
$$T = \frac{2\pi}{\dot{\psi}} = 6.84 \text{ Myr}$$

Due to the Sun's pull on the equatorial bulge of Saturn, the axis of rotation of Saturn wobbles with a period some ~ 7 Myr.

What caused Saturn's obliquity to increase from $\sim 0^\circ$ to 26.7° ?

- Can't be tides because the Sun is too far and Titan not massive enough.
- Large spin $+$ Large mass $\rightarrow$ Giant impact inefficient.
- Saturn's moons merely change the equinox precession period.

Taking into account Saturn's moons



Actual period of Saturn's equinox precession

$$T = 1.8 \text{ Myr}$$

A secular resonance between a frequency in the Solar system and the frequency of the wobble of Saturn ?

Saturn wobbles at a rate

$$\dot{\psi} \approx -0.74''/\text{yr}$$

Is there such a frequency in the Solar system ?

Yes there is

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Frequencies in the Solar system (Saillenfest et al. 2021 from Laskar 1990)

k	Identification$^{(*)}$	ν_k ($''$ yr^{-1})	$S_k \times 10^8$	$\phi_k^{(0)}$ (°)
1	s_5	0.00000	1 377 395	107.59
2	s_6	−26.33023	785 009	127.29
3	s_8	−0.69189	55 969	23.96
4	s_7	−3.00557	39 101	140.33
5	$g_5 - g_6 + s_7$	−26.97744	5889	43.05
6	$2g_6 - s_6$	82.77163	3417	128.95
7	$g_5 + g_6 - s_6$	58.80017	2003	212.90
8	$2g_5 - s_6$	34.82788	1583	294.12
9	s_1	−5.61755	1373	168.70
10	s_4	−17.74818	1269	123.28
11	$-g_5 + g_7 + s_6$	−27.48935	1014	218.53
12	$g_5 - g_7 + s_6$	−25.17116	958	215.94
13	$g_5 - g_6 + s_6$	−50.30212	943	209.84

Secular resonance between the wobble of Saturn and the ascending node of Neptune

$$\dot{\psi} = \alpha \cos \varepsilon$$

Precession constant

$$\alpha = -\frac{3n^2MR^2}{2C\dot{\theta}}J_2 \text{ for Saturn}$$
$$\alpha = \frac{2Mgl}{C\dot{\theta}} \text{ for the top}$$

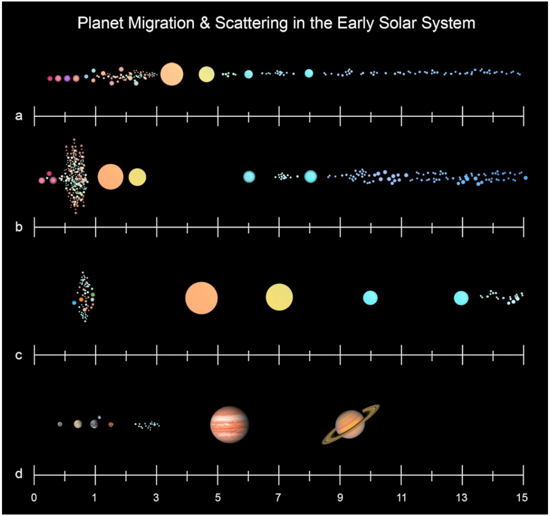
If the frequency s_8 of the ascending node of Neptune was larger in the past, or if the precession constant α of Saturn's wobbling was smaller, Saturn could have been trapped by Neptune.

First paradigm (Ward & Hamilton 2004)

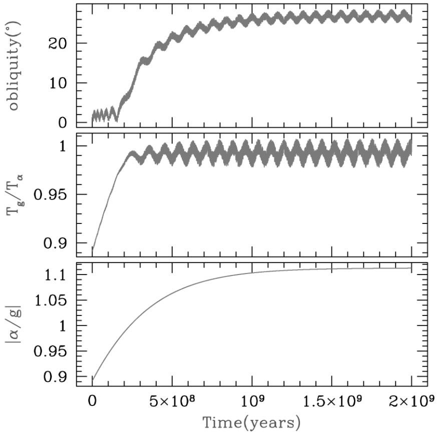
A late migration of Neptune decreased s_8 to ensure the capture in resonance

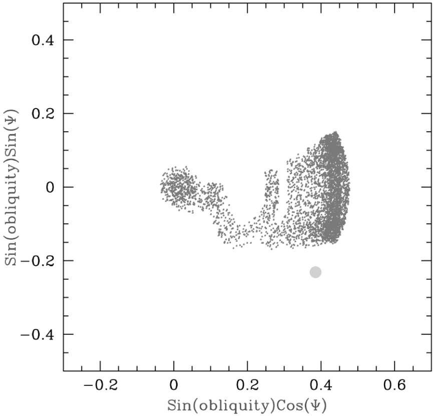
Grand tack

Fernandez 1984
Malhotra 1993
Morbidelli+ 2005
Gomes+ 2005
Tsiganis+ 2005
Walsh+ 2011

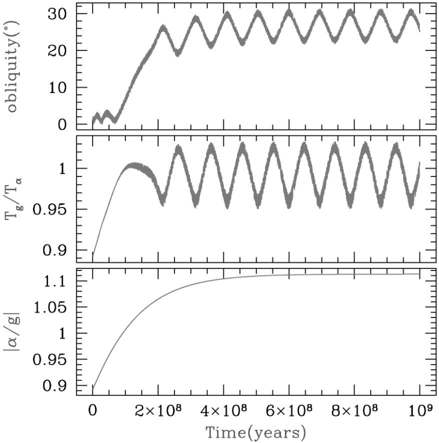


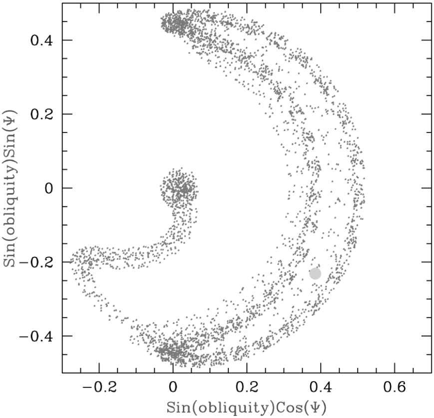
Slow capture scenario into $s_8 = \dot{\psi}$ resonance (Hamilton & Ward, 2004)



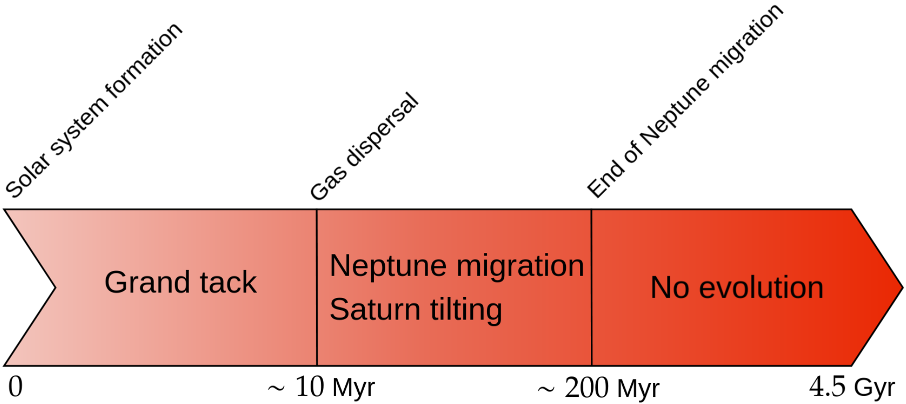


Fast capture scenario into $s_8 = \dot{\psi}$ resonance (Hamilton & Ward, 2004)





Conclusion of the paradigm of Ward & Hamilton, 2004

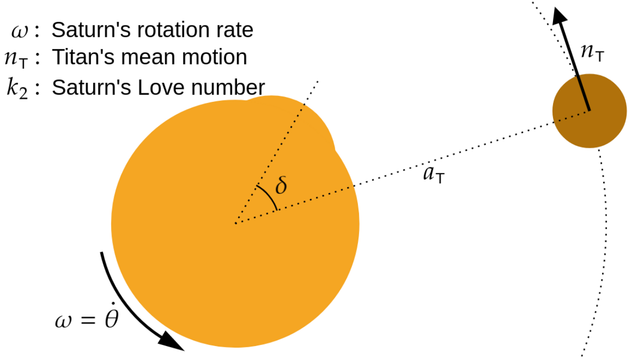


Second paradigm (Saillenfest, Lari, Boué, Courtot, 2021)

There is a problem ... That didn't happen like that ...

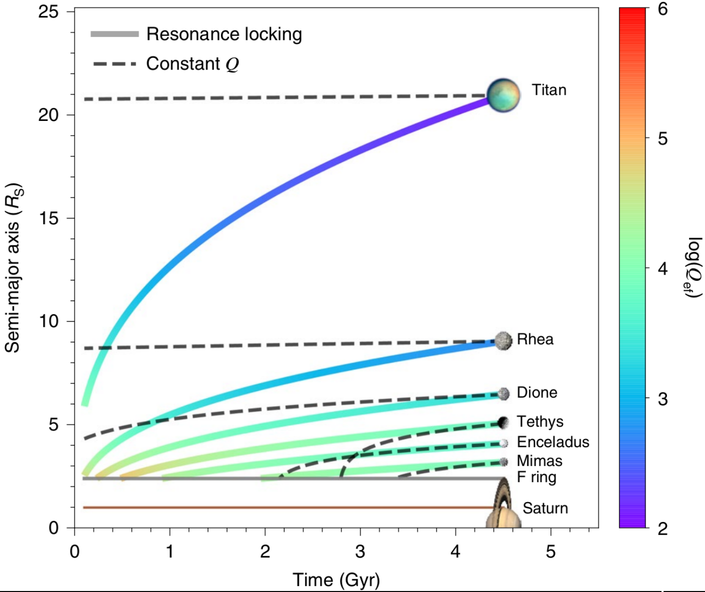
- Titan quickly migrates away from Saturn (Lainey et al. 2020)
- Effective precession constant $\alpha = \frac{3}{2}\frac{n^2}{\dot{\theta}}\frac{J_2'}{\lambda'}$ depends on J_2'
- Effective equatorial bulge $J_2' = J_2 + \frac{1}{2}\frac{m_\mathsf{T}}{M_\mathsf{s}}\frac{a_\mathsf{T}^2}{R_\mathsf{s}^2}$ depends heavily on a_T
- As Titan migrates, a_T increases and so does α
- In the early Solar system, Saturn used to wobble much slower
- The system was far from the resonance $s_8 = \dot{\psi}$
- The tilt of Saturn was due to the migration of Titan, not to the migration of Neptune

Why does Titan migrate ?

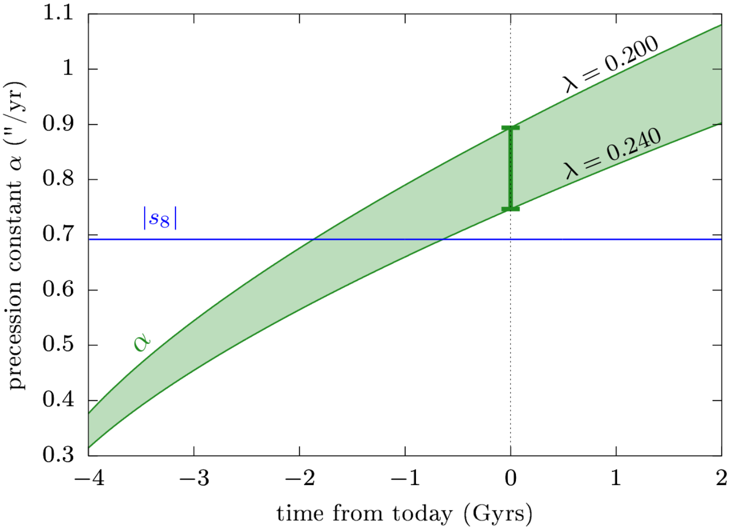


$$\frac{\dot{a}_{\mathsf{T}}}{a_{\mathsf{T}}} = 3\frac{k_2}{Q}\frac{m_{\mathsf{T}}}{M_{\mathsf{s}}}\left(\frac{R_{\mathsf{s}}}{a_{\mathsf{T}}}\right)^5 n_{\mathsf{T}} \qquad Q = \frac{1}{\tan\delta}$$

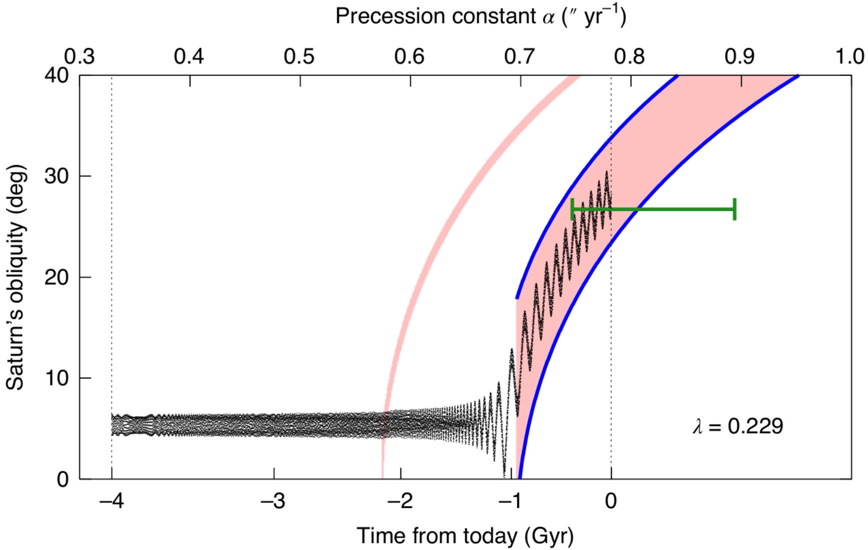
Fast tidal migration of Saturnian moons
(Lainey et al. 2020)



Time evolution of α (Saillenfest et al. 2021)

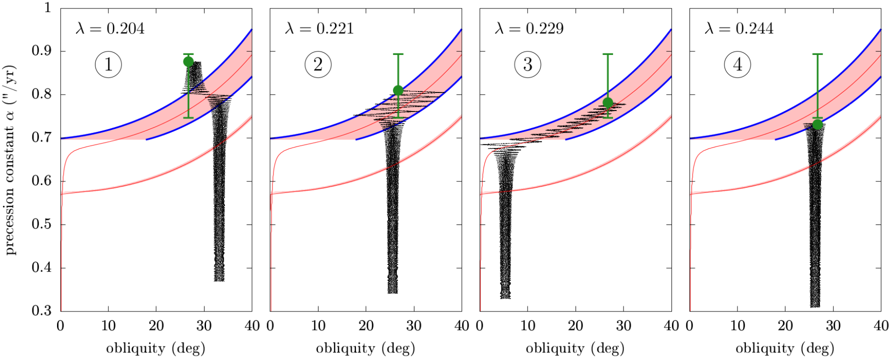


Resonance capture (Saillenfest et al. 2021)



Large uncertainty on C

$$0.20 \leq \frac{C}{MR^2} \leq 0.24 \quad \Rightarrow \quad 0.747''/\text{yr} \leq \alpha \leq 0.894''/\text{yr}$$



Extracted from Saillenfest et al. 2021

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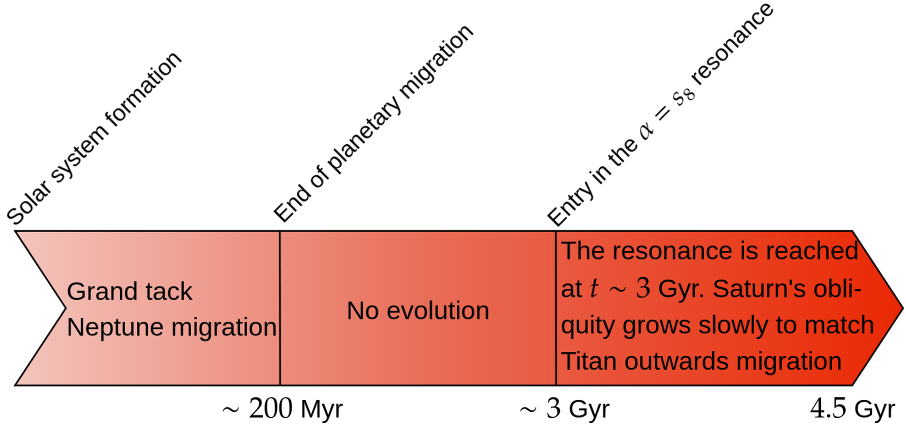
Conclusion of the paradigm of Saillenfest et al. 2021



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Third paradigm (Wisdom et al. 2022)

There is still a problem ...

- Second paradigm suggests that the system is still in the resonance
- But... The system is currently outside the $\dot{\psi} = s_8$ resonance
- Something happened that made the system exit the resonance

Due to the large uncertainty on C, Saillenfest et al. did not know if the system was still in the resonance.

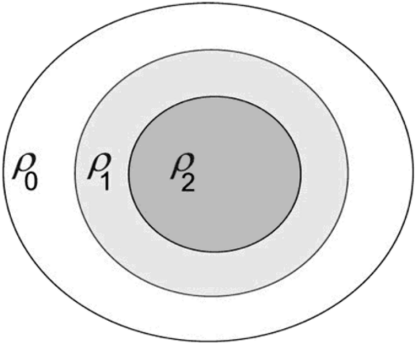
They assumed it was because it is was suggested by their paradigm.

How do Wisdom et al. know the system is outside the resonance?

- Cassini spacecraft orbited Saturn for many years
- Gravitational field outside Saturn is well constrained (Iess et al. 2019)
- They use the knowledge of the gravitational field to constrain C

How to constrain the moment of inertia C?

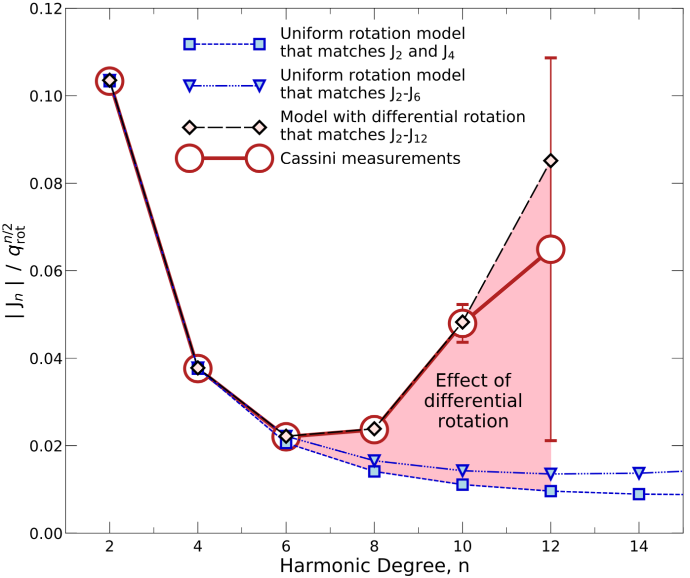
Wisdom et al. assume Saturn is a superposition of concentric MacLaurin spheroid (Hubbard 2013)



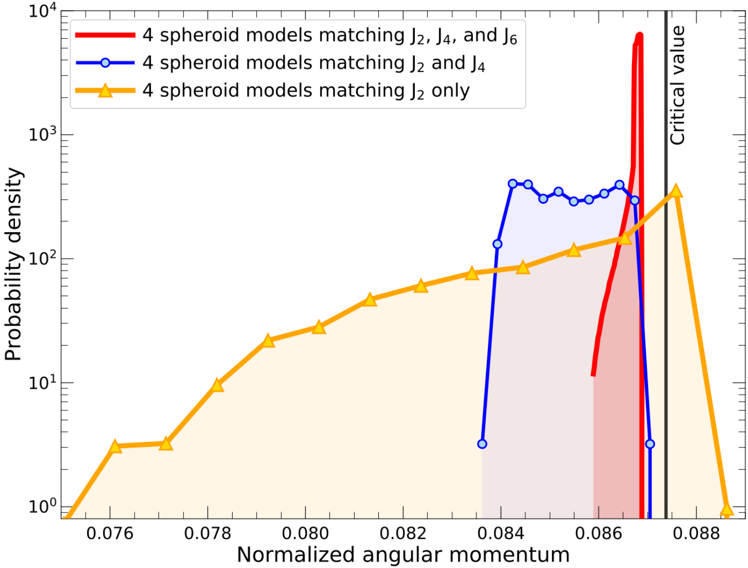
Extracted from Hubbard (2013)

- Spheroids' densities and radii chosen to match Saturn's gravitational field
- The moment of inertia is then computed by integrating over Saturn

Fit of Wisdom et al. 2022



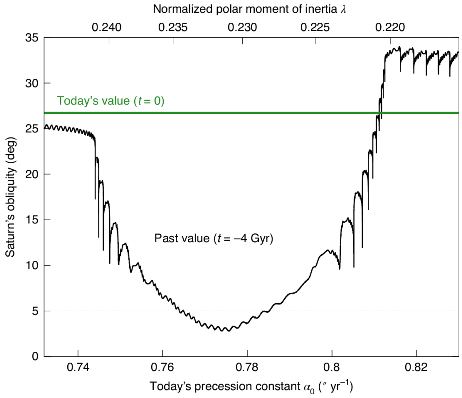
Saturn is not in the $\dot{\psi} = s_8$ resonance



Final value of λ retained by Wisdom et al. 2022

$$0.2179 \leq \lambda \leq 0.2188$$

Range of Saillenfest et al. 2021

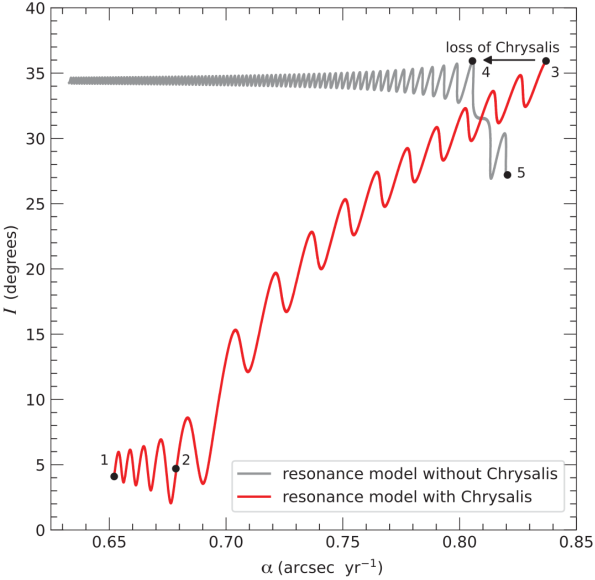


$$\varepsilon(t = -4 \text{ Gyr}) \leq 10^\circ \quad \Rightarrow \quad 0.225 \leq \lambda \leq 0.237$$

Paradigm of Wisdom et al. 2022

- Such a closeness to the secular resonance is unlikely to be coincidental
- The resonance was encountered, and then left
- They propose that Saturn used to have a now disappeared satellite, named *Chrysalis*
- The $3:1$ mean motion resonance between migrating Titan and Chrysalis was encountered.
- This resonance with Titan excited Chrysalis eccentricity so much that it made a grazing approach of Saturn
- Tidal forces broke Chrysalis up and rings were formed.
- When Chrysalis broked up, J_2' suddenly dropped, and so did α, which allowed Saturn to escape the Neptune resonance.

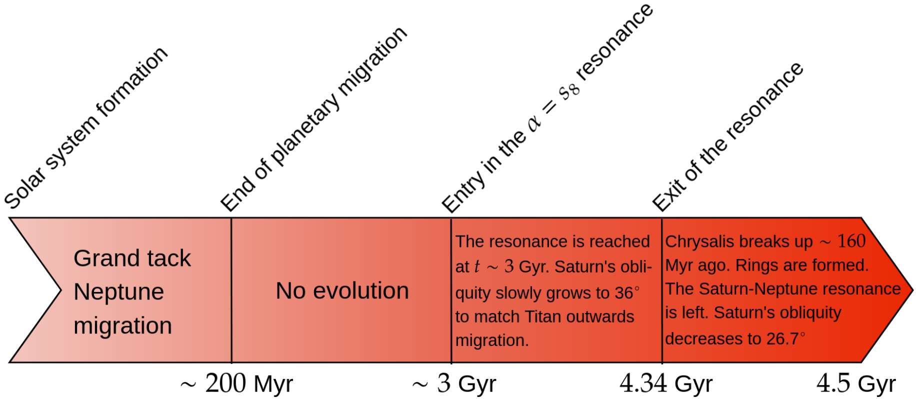
Saturn's obliquity with Chrysalis



This third paradigm of Wisdom et al. 2022 gives an explanation to

- The current obliquity of Saturn
- Why Saturn is so close to, but outside, the Neptune resonance
- The presence of young rings around Saturn
- The quite large 0.0288 eccentricity of Titan (through the $3:1$ mean motion resonance with Chrysalis)
- The presence of short-lived methane in Titan atmosphere (tidal heating due to increased eccentricity)

Conclusion of the paradigm of Wisdom et al. 2022



Final conclusion

- The journey towards understanding Saturn's obliquity was bumpy
- I hope you liked the ride !
- It might not be over ...

Thank you for your attention