Tilting Uranus via the migration of an ancient satellite

Saillenfest et al. 2022

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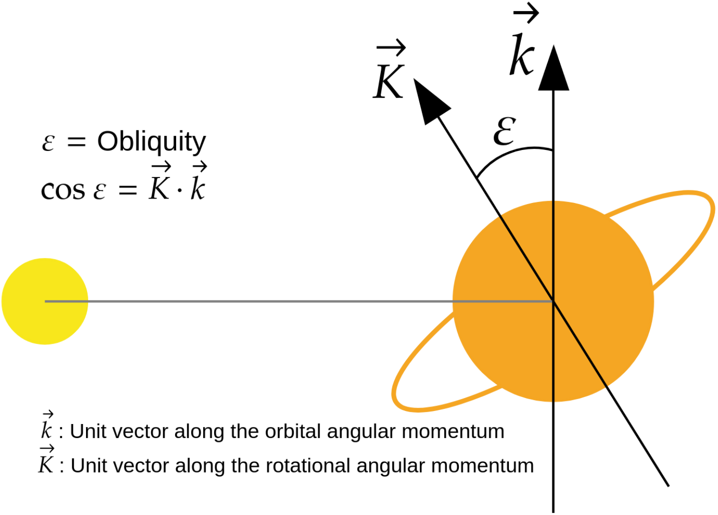
What is the obliquity ?



ε = Obliquity

$\cos\varepsilon = \vec{K} \cdot \vec{k}$

$\vec{k}$: Unit vector along the orbital angular momentum

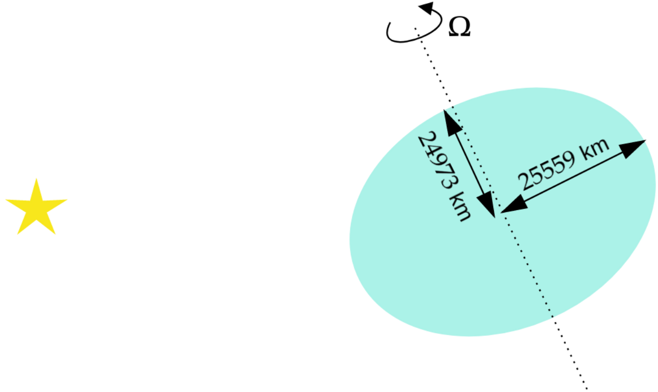
$\vec{K}$: Unit vector along the rotational angular momentum

Obliquities in the Solar system

Planet	Obliquity
Mercury	$0.034°$
Venus	$177.4°$
Earth	$23.44°$
Mars	$25.19°$
Jupiter	$3.13°$
Saturn	$26.73°$
Uranus	$97.77°$
Neptune	$28.32°$

- Laskar & Robutel (1993):
  The inner planets' obliquities were chaotic at some point $\rightarrow$ Not primordial.
- Planets are expected to form with near-zero obliquity.
- The outer planets' obliquities should be primordial but only Jupiter's is
- Why is Uranus lying on its side ?

Equatorial bulge



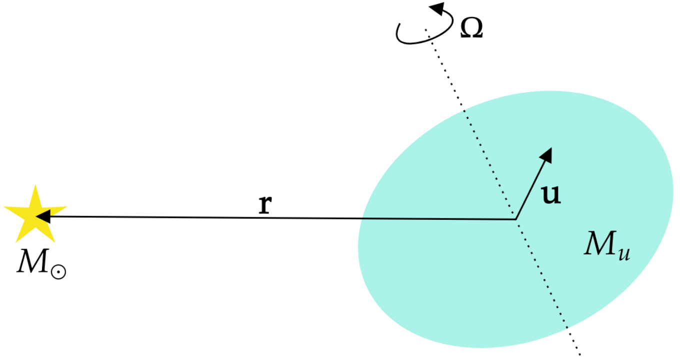
- Uranus is slightly flattened due to its rotation → equatorial bulge
- The pull of the Sun and of the satellite on this bulge yields a torque

How does Uranus react to this torque ?

Colombo model (Colombo, 1966)

- I present a simple Hamiltonian version of the Colombo model.
- The Hamiltonian is written $\mathcal{H} = \mathcal{T} + \mathcal{U}$ where
- $\mathcal{T}$ is the kinetic energy of rotation of Uranus on itself
- $\mathcal{U}$ is the potential energy associated with the interaction between the bulge and the Sun.
- To only retain the long-term evolution of the rotation, $\mathcal{U}$ is averaged over one orbital period of Uranus.

What about the potential energy ?

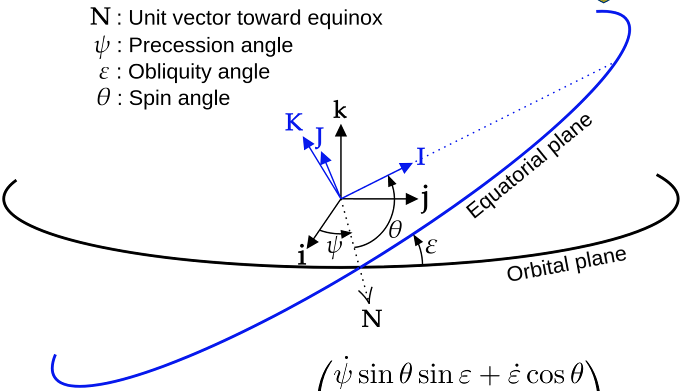


$$\mathcal{U} = -\int_{♄} \frac{\mathcal{G} M_{\odot}}{|\mathbf{r} - \mathbf{u}|} dM + \frac{\mathcal{G} M_{\odot} M_{\mathsf{u}}}{r}$$

$$\langle \mathcal{U} \rangle := \frac{1}{T} \int_0^T \mathcal{U} dt = -\frac{\mathcal{G} M_{\odot} (C - A)}{4 a_{\odot}^3 \left(1 - e_{\odot}^2\right)^{3/2}} \left(3 \cos^2 \varepsilon - 1\right)$$

C and A are the polar and equatorial moment of inertia, respectively

What about the kinetic energy ?



$$\mathbf{\Omega} = \dot{\psi}\mathbf{k} + \dot{\varepsilon}\mathbf{N} + \dot{\theta}\mathbf{K} = \begin{pmatrix} \dot{\psi}\sin\theta\sin\varepsilon + \dot{\varepsilon}\cos\theta \\ \dot{\psi}\cos\theta\sin\varepsilon - \dot{\varepsilon}\sin\theta \\ \dot{\psi}\cos\varepsilon + \dot{\theta} \end{pmatrix}_{\mathbf{I},\mathbf{J},\mathbf{K}}$$

$$\mathcal{T} = \frac{1}{2}\,{}^t\mathbf{\Omega}\mathcal{I}\mathbf{\Omega} \qquad \mathcal{I} = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix}$$

Generalized coordinates and momenta

$$\mathbf{q} = \begin{pmatrix} \psi \\ \varepsilon \\ \theta \end{pmatrix} \qquad \mathbf{p} := \frac{\partial \mathcal{T}}{\partial \dot{\mathbf{q}}}$$

Hamilton equations

$$\dot{\mathbf{q}} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}} \qquad \dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{q}}$$

Secular variations of the rotation of Saturn

$$\dot{\theta} = \text{Cste} \;\rightarrow\; \text{Length of day} = \text{Cste}$$

$$\dot{\varepsilon} = 0 \;\rightarrow\; \text{No change in obliquity}$$

$$\dot{\psi} = -\frac{3}{2}\lambda^{-1}\frac{n^2}{\dot{\theta}}\left(1 - e_{\odot}^2\right)^{-3/2} J_2 \cos\varepsilon$$

where n is the mean motion of Uranus, $e_{\odot}$ the eccentricity of its orbit, $J_2 = \left(C - A\right)/M_{\mathsf{u}}R_{\mathsf{u}}^2$ its second zonal harmonic and $\lambda = C/M_{\mathsf{u}}R_{\mathsf{u}}^2$ is the normalized polar moment of inertia of Uranus.

Precession of the equinox

The pull of the Sun on the equatorial bulge yields a wobble of the axis of rotation with frequency

$$\dot{\psi} = -p\cos\varepsilon = -\frac{3}{2}\frac{n^2}{\dot{\theta}}\left(1 - e_{\odot}^2\right)^{-3/2}\frac{J_2}{\lambda}\cos\varepsilon$$

For Uranus, the corresponding period is

$$T = \frac{2\pi}{\dot{\psi}} = 22.8 \text{ Myr}$$

But what about the contribution from potential satellites ?

Contribution from a satellite There are two ways satellites can affect the frequency of the wobble

- If the satellite is far away and not in the equatorial plane, it pulls on the bulge like the Sun does.
- If it is a close-in satellite, it is locked to the equatorial plane and artificially contributes to increase the J_2 of the planet. $\rightarrow$ It gives leverage to the Sun.

Correction of the equinox precession with a satellite

$$\dot{\psi} = -p \left(\cos \varepsilon + \eta \frac{a^2}{r_M^2} \frac{\sin(2\varepsilon - 2I_L)}{2 \sin \varepsilon} \right)$$

where $\eta = m_{\mathsf{s}} r_M^2 / (2 M_{\mathsf{u}} J_2 R_{\mathsf{u}}^2)$ is the dimensionless mass parameter, $r_M \approx 53 R_{\mathsf{u}}$ and I_L is a inclination whose value depends on whether the satellite is in the close-in or far away regime.

The Colombo model applied to Uranus does not predict obliquity variations.

What caused Uranus' obliquity to increase from $\sim 0°$ to $97.77°$?
Previous studies (*e.g.* Slattery et al. 1992) suggest that giant impacts during the late heavy bombardment tilted Uranus.

Why is this unlikely ?

- Uranus and Neptune have strikingly similar masses, radii and spin rates, but very different obliquities ($98°$ vs $28°$).
- They also have similar atmospheric dynamics and magnetic fields.
- One does not expect such similarities from random collisions.

We would expect more diversity if Uranus and Neptune's final formation stages were determined by random collisions.

Let us look for a smoother process $\cdots$ A resonance of course !

Planetary motion in the solar system $\rightarrow$ Forcing on Uranus' spin dynamics.

Fourier decomposition of Laskar 1990

$$e_\odot e^{i\varpi_\odot} = \sum_{k\in\mathbb{Z}} E_k e^{i\left(\mu_k t+\theta_k^{(0)}\right)} \qquad \sin(i_\odot/2)e^{i\Omega_\odot} = \sum_{k\in\mathbb{Z}} S_k e^{i\left(\nu_k t+\phi_k^{(0)}\right)}$$

- $e_\odot$: Uranus' eccentricity
- $\varpi_\odot$: Uranus' longitude of periapsis
- $i_\odot$: Uranus' inclination on the ecliptic
- $\Omega_\odot$: Uranus' longitude of the ascending node

μ_k and ν_k are the fundamental frequencies of the Solar system.

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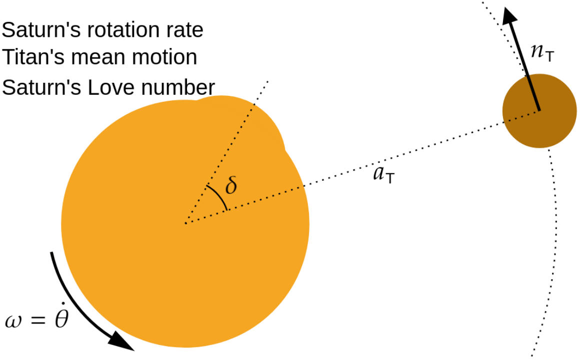
Uranus frequencies (Saillenfest et al. 2022 from Laskar 1990)

k	Identification$^{(*)}$	ν_k ($''$ yr^{-1})	$S_k \times 10^9$	$\phi_k^{(0)}$ ($^{\circ}$)
1	s_5	0.00000	13 773 646	107.59
2	s_7	−3.00557	8 871 413	320.33
3	s_8	−0.69189	563 042	203.96
4	s_6	−26.33023	347 710	307.29
5	$-g_5 + g_6 + s_6$	−2.35835	299 979	224.75
6	$-g_5 + g_7 + s_7$	−4.16482	187 859	231.66
7	$g_5 - g_7 + s_7$	−1.84625	182 575	224.56
8	$-g_7 + g_8 + s_8$	−3.11725	59 252	146.97
9	$g_6 - g_7 + s_6$	−1.19906	25 881	313.99
10	$2g_5 - s_7$	11.50319	18 941	101.01
11	$g_5 + g_7 - s_7$	10.34389	11 930	11.68
12	$g_5 - g_6 + s_7$	−26.97744	10 362	225.10
13	s_1	−5.61755	10 270	348.70
14	$-g_5 + g_6 + s_7$	20.96631	7346	237.78
15	$g_7 - g_8 + s_7$	−0.58033	5474	197.32

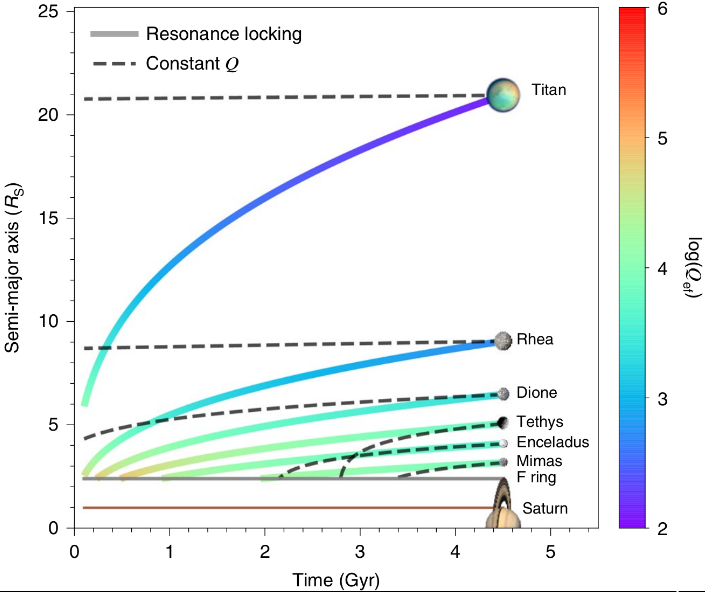
Why do satellite migrate ?

ω : Saturn's rotation rate
n_T : Titan's mean motion
k_2 : Saturn's Love number



$$\frac{\dot{a}_\mathsf{T}}{a_\mathsf{T}} = 3\frac{k_2}{Q}\frac{m_\mathsf{T}}{M_\mathsf{s}}\left(\frac{R_\mathsf{s}}{a_\mathsf{T}}\right)^5 n_\mathsf{T} \qquad Q = \frac{1}{\tan\delta}$$

Fast tidal migration of Saturnian moons
(Lainey et al. 2020)

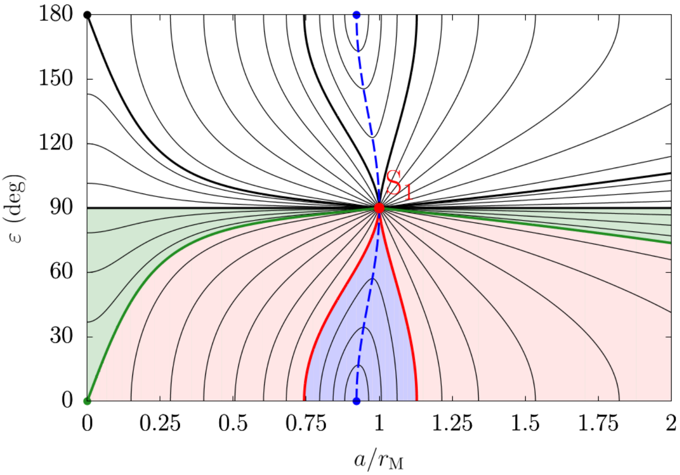


Mechanism of tilting

$$\dot{\psi} = -p\left(\cos\varepsilon + \eta\frac{a^2}{r_M^2}\frac{\sin(2\varepsilon - 2I_L)}{2\sin\varepsilon}\right) = \nu_k \approx \text{Cst}$$

- If the semi-major axis a of the ancient satellite increases due to tides
- Then $\cos\varepsilon$ must decrease to maintain the relation.
- The level lines of $\dot{\psi}$ are followed during the tilting

Level lines of $\dot{\psi}$



Pink region

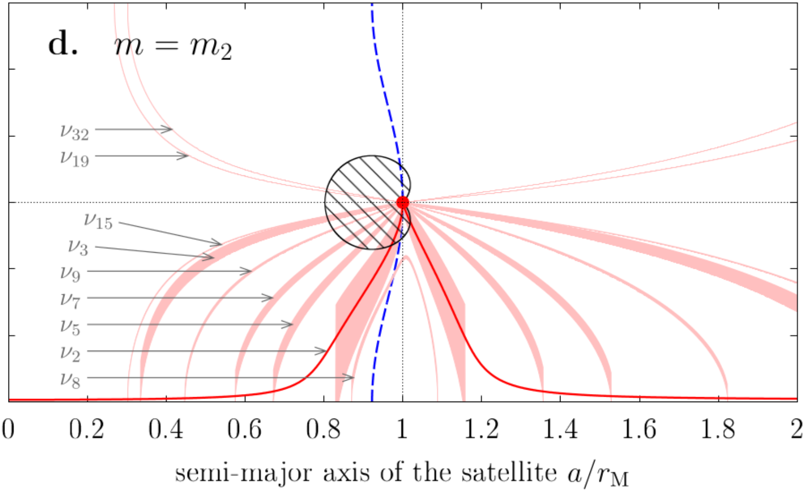
$$p \leq |\nu_k| \leq p\frac{\eta}{2}$$

Minimal mass for a satellite able to tilt Uranus

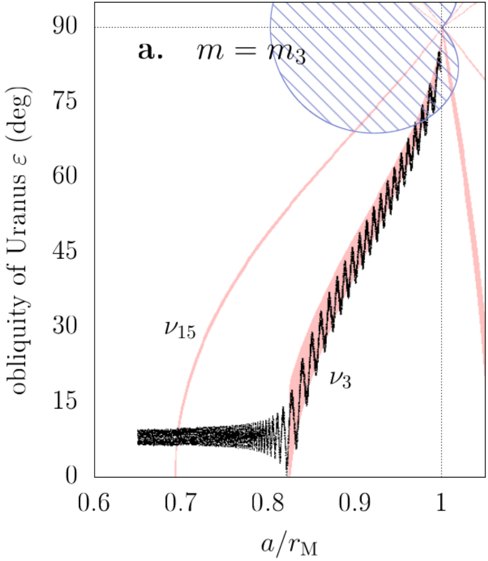
$$|\nu_k| \leq p\frac{\eta}{2} = \frac{p}{4}\frac{m_{\mathsf{s}}}{M_{\mathsf{u}}}\frac{r_M^2}{J_2 R_{\mathsf{u}}^2}$$

k	Identification	ν_k ($''$ yr^{-1})	$T_{\mathrm{lib}}^{(\mathrm{sep})}$ (Myr)	m_{min}/M ($\times 10^{-5}$)	m_k/M ($\times 10^{-5}$)
15	$g_7 - g_8 + s_7$	−0.58033	3021	35	36
3	s_8	−0.69189	115	41	44
9	$g_6 - g_7 + s_6$	−1.19906	519	71	79
7	$g_5 - g_7 + s_7$	−1.84625	92	110	129
5	$-g_5 + g_6 + s_6$	−2.35835	51	141	172
2	s_7	−3.00557	4	179	233
8	$-g_7 + g_8 + s_8$	−3.11725	115	186	244
6	$-g_5 + g_7 + s_7$	−4.16482	40	248	368

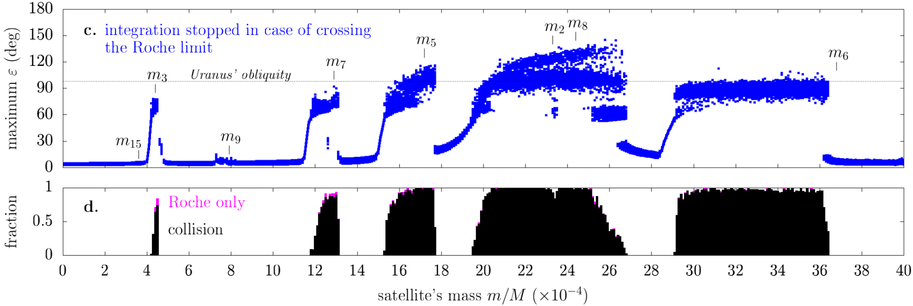
Resonance $\dot{\psi} = s_7$ $(m_2 = 2.3 \times 10^{-3} M_{\mathbf{u}})$



Example of tilting to a large obliquity

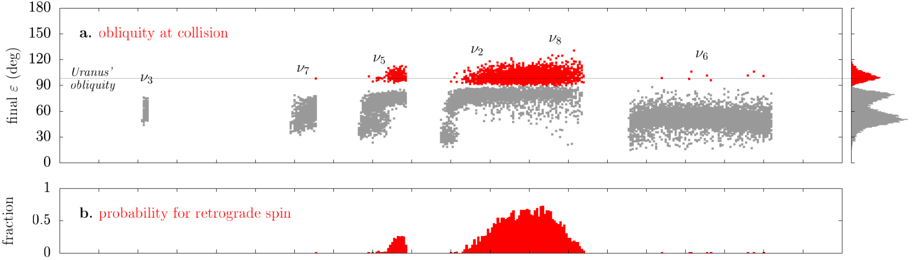


Maximum obliquity as a function of m_s



Resonances 2, 5, 6 and 8 look to be potential candidates

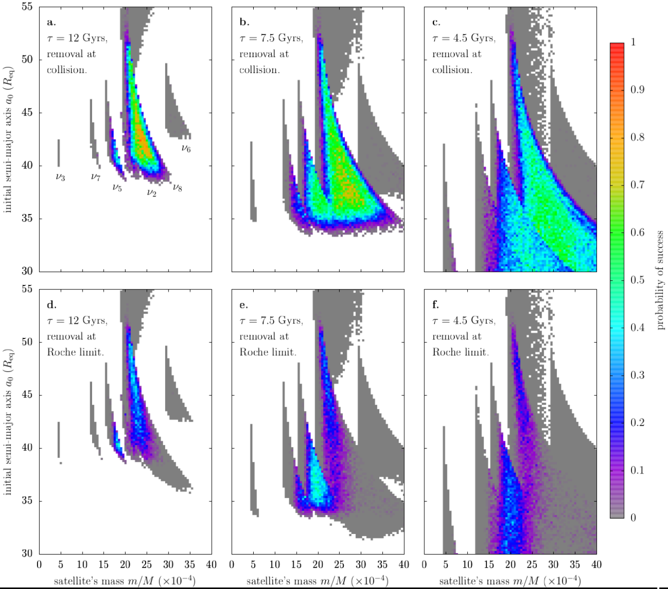
Obliquity at collision as a function of m_s



Resonances 2 and 8 not only can reproduce Uranus' obliquity, but that is their most likely outcome !

- Resonance 2 : $\dot{\psi} \approx s_7$: Forcing due to Uranus' ascending node
- Resonance 8 : $\dot{\psi} \approx s_8 + g_8 - g_7$: Forcing due to Neptune's ascending node and Neptune and Uranus' periapsis.

Probability of success



Conclusion

- Provided that enough massive ancient satellite existed, this is a likely mechanism
- The tilting could also have been due to several satellite

Thank you for your attention